

Test 7: Section 8

1. $6700 = 100(6k + 7)$

Divide both sides of the equation by 100 to get:

$$67 = 6k + 7$$

Subtract 7 from both sides of the equation to get:

$$60 = 6k$$

Divide both sides by 6 to get:

$$10 = k.$$

2. 3 more than n is negative. Thus, $n+3$ is a negative number.
5 more than n is positive. Thus, $n+5$ is positive.
You can do this by testing out the answers. B works. $-4+3 = -1$, so $n+3$ is negative.
 $-4+5 = 1$, so $n+5$ is positive.

Or you can use an algebraic approach:

$n+3 < 0$ (because $n+3$ is negative)
subtract 3 from both sides to get:

$$n < -3$$

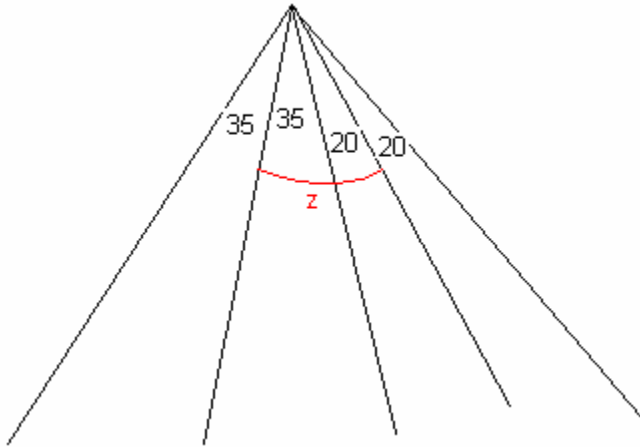
$n+5 > 0$ (because $n+5$ is positive).
Subtract 5 from both sides to get:

$$n > -5$$

Thus, $n < -3$ and $n > -5$. The only answer choice that works is B, $n = -4$.

3. $x = 70$. The dotted line that bisects it cuts into two equal angles of 35 each. To “bisect” means to “cut in half”.

$y = 40$. The line that bisects it cuts it into two equal angles. Each angle is a 20 degree angle. Thus, we have this so far:



z is marked in red. It is the sum of the 35 and 20 degree angle. Thus, its measure is $35+20 = 55$.

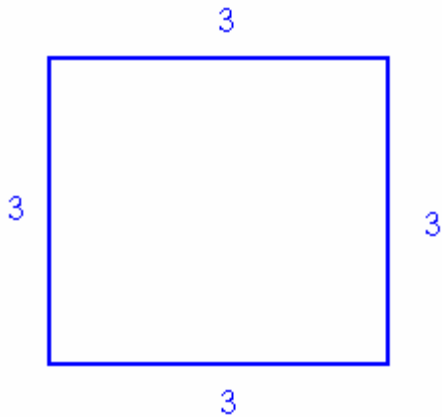
4. probability = $\frac{\text{part}}{\text{whole}}$

The probability of getting an apple is $\frac{2}{5}$. That means we could have 2 pieces of apple out of 5 total pieces of fruits. Or we could have 4 pieces of apple and 10 total pieces. Or 200 pieces of apple and 500 total pieces. The only restriction is that the total number of pieces must be a multiple of 5.

20, 35, 70, and 80 are all multiples of 5. 52 is not. Thus the correct answer is C. Remember that the question asks what number will NOT work.

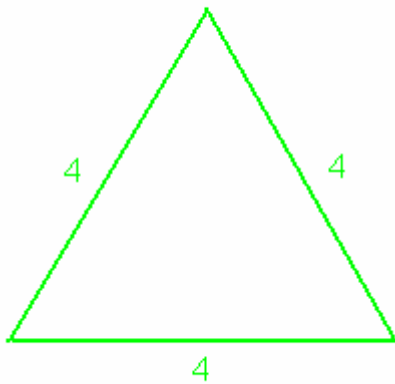
5. The perimeter is the distance around the figure. It is the sum of all the side lengths.

A square has sides of length 3. Here is a picture:



The perimeter is $3+3+3+3 = 12$.

All the sides of an equilateral triangle are equal. Also, the perimeter must be 12, since it has to have the same perimeter as the square. Thus, each side is $12/3 = 4$. Here is a picture:



6. x is -1 . $(-1)^{\text{(an even power)}} = 1$. For example, $(-1)^2 = (-1)*(-1) = 1$, and $(-1)^{16} = 1$.

On the other hand -1 raised to an odd number is -1 .

For example, $(-1)^3 = (-1)(-1)(-1) = -1$

Thus, B and D are positive numbers. A, C, E are negative numbers. Any positive number is obviously bigger than any negative number.

Now the question which is bigger, B or D?

$$B = 4k(-1)^2 = 4k*1 = 4k$$

$$D = 8k(-1)^4 = 8k*1 = 8k$$

Since k is positive, $8k > 4k$. The answer is D.

7. She does swimming, then cycling, then running.

The runs faster than she swims. The cycles faster than she runs. Thus, she cycles fastest, runs second fastest, and swims the slowest.

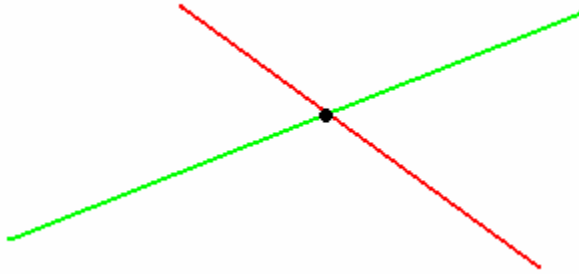
Thus, here is the order

1. Swimming (slow)
2. Cycling (Fastest)
3. Running (medium)

The speed is reflected in the slope. If she is moving faster, the graph has a steeper slope. Why? Because if she goes faster, she covers more distance (y axis) in the same amount of time (x axis).

Thus, the graph should start out with a small slope, then have a really steep slope, then a medium slope. The answer is E.

8. If a point is on the intersection of two graphs, it must be a point on both graphs. Here's an illustration of this principle:



The intersection point is on the red line and the green line.

Thus $(\sqrt{6}, k)$ is on both $y=x^2 - 7$ and on $y=-x^2+j$

We will plug the point into the first equation to get k:

When $x=\sqrt{6}$, $y=k$. Plugging these values into the equation $y=x^2 - 7$ gets you:

$$k = (\sqrt{6})^2 - 7$$

$$k = 6-7$$

$$k = -1.$$

Now we can plug $x=\sqrt{6}$ and $y=k=13$ into the other equation to solve for j:

$$y = -x^2+j$$

$$-1 = -(\sqrt{6})^2 + j$$

$$-1 = -6 + j$$

Add 6 to both sides of the equation to get:

$$5 = j$$

9. The easiest way is to just test the answer choices. The correct answer is 4.
Plug in 3 for x in $|2-x|$

$$|2-4| = |-2| = 2$$

Alternately, you can do it algebraically.

$|2-x| < 3$. Thus:

$$2-x < 3 \text{ AND } 2-x > -3$$

$$-x < -1 \text{ AND } -x > -5$$

When you multiply by a negative number, remember to reverse the inequality sign:

$$x > 1 \text{ AND } x < 5$$

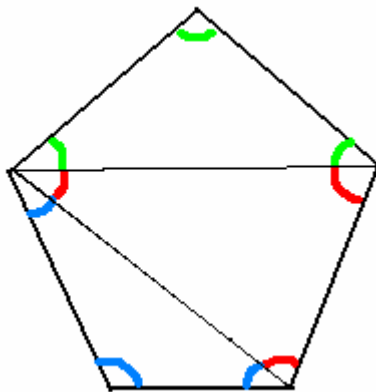
The only answer choice that is less than 5 is 4. Thus, the answer is A.

10. The angles in a pentagon add up to 540. There are several ways to know this.

One is the formula (sum of angles) = $(n-2)180$, where n is the number of sides. The polygon in the picture has 5 sides (making it a pentagon). Thus,

$$(\text{sum of angles}) = (5-2)180 = 3(180) = 540$$

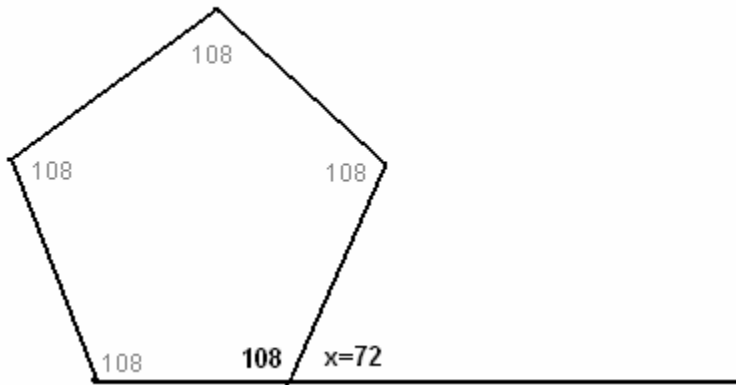
OR, you can divide the pentagon up into triangles to get something like this:



There are three triangles here. Each triangle's internal angles add up to 180, and all the angles combined are the same as the angles of the pentagon. Thus, the sum of the angles of the pentagon is $3 \cdot 180 = 540$.

The sum of the angles is 540. All angles are equal, according to the problem. Thus, each angle is $540/5 = 108$.

Now we can find x . If two angles form a straight line, their sum is 180. This is because a straight line is a 180 degree angle. Thus:



$$108+x = 180$$

subtract 108 from both sides of the equation to get:

$$x=72$$

11. x = length of the actual tool.

The length of the drawing is $\frac{3}{8}$ of the length of the actual tool . Thus,

$$\text{length of drawing} = \left(\frac{3}{8}\right)x$$

The length of the drawing is 6, according to the question. Thus:

$$\frac{3}{8} * x = 6$$

Multiply both sides by $\frac{8}{3}$ to get:

$$\frac{8}{3} * \frac{3}{8} * x = 6 * \frac{8}{3}$$

Cancel and then multiply to get:

$$x = 16$$

12. Only even numbers can be divided by two to give integers. For example, $4/2 = 2$, $8/2 = 4$, $14/2 = 7$.

An odd number divided by 2 will not give an integer. For example, $7/2 = 3.5$, which is not an integer. $17/2 = 8.5$, which is also not an integer. (An integer is a positive or negative whole number, or 0. 0, 1, 53, -9 are all integers).

Thus, $x+3$ must be even.

Here are the rules:

The sum of two odd numbers is even.

The sum of an odd and an even is odd.

The sum of 2 even numbers is even.

3 is odd. For $x+3$ to be even, x must also be odd. Thus, the answer is E.

13. The x coordinate of Q is 3; it has the same x coordinate as P. The y coordinate is 3. Why? The center is halfway across the diameter. Thus, the Q is half as high as P; since the y coordinate of P is 6, the y coordinate of Q is 3. Thus, the coordinates of Q are (3,3)

Using similar reasoning, we can see that the x coordinate of S is 11, and the y coordinate of S is $10/2 = 5$. Thus, the coordinates of S are (11,5)

We need to find the slope between Q, which is (3,3), and S, which is (11,5).

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{11 - 3} = \frac{2}{8} = \frac{1}{4}$$

14. Here's a bit of background discussion:

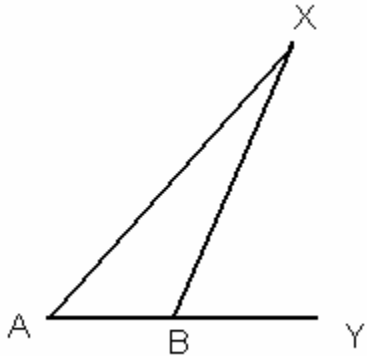
Multiples of 5 are 5 units apart. For example, 5, 10, 15... are all 5 units apart. Thus, two multiples of 5 are either 5 units apart or some multiple of 5 units apart. For example, 5 and 20 are 15 units apart; 15 is a multiple of 5.

The same applies to multiples of 6, 7 or any other number

Now let's get to the problem. p is a factor of n+3 and n+10.

Here's how to find p. First notice that (n+3) and (n+10) are 7 units apart. If they are multiples of the same number, that number must be 7. For example, 21 and 28 are 7 units apart, and are both multiples of 7. Multiples of no other number can be 7 units apart, because no other number goes into 7 (7 is a prime number). The only exception is 1, but the problem specifically states that p is greater than 1.

15. The following is a drawing of XAY and XBY, based on the picture in the book:



Which angle is smaller, XAY or XBY? XAY is smaller. Notice that since A is farther away from Y, it makes a smaller angle.

Using this logic, we just need to find which point is farthest from Y. This point will make the smallest angle.

Looking at the picture, we can see that D is farthest away from Y of all the points. Thus, XDY is the smallest angle.

(You can prove that D is the farthest point by using the Pythagorean theorem. However this is not necessary – you can easily tell by looking at the diagram.)

16. When you look at a question like this, you should ask why on earth are they asking for “ $x^2y - y^2x$ ”. There has got to be some reason. We have some clues: they tell us the value of xy and the value of $x-y$.

Here’s the reason:

$$xy(x-y) = x^2y - y^2x$$

We know that $xy=7$ and $x-y = 5$. Thus, the above equation becomes:

$$\begin{aligned} 7(5) &= x^2y - y^2x \\ 35 &= x^2y - y^2x \end{aligned}$$